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The Mathematical Basis of the Phenomenal World

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ABSTRACT

In the *Critique of Pure Reason* Immanuel Kant said that cognition (objective perception) is acquired in the unity of sensibility (the receptivity of the mind to receive empirical representations of things, which yields intuitions) and the understanding (in which concepts – general representations of things – arise) and is mediated by the imagination. Here, it is shown that numbers, either pure or denominate, are cognized in the synthesis of intuition and mathematical concept, and that the phenomenal world of the cognizer is shaped accordingly. Any number can be related to any other number through a *general* mathematical formula conceived by the cognizer for the purpose. The judgment of the cognizer is manifest in the *specifics* of the mathematical relationship established between the two numbers in cognition. If the cognized number is the numerical value of a physical constant, then in the (consistent) phenomenal world it will always have been of the value found in cognition, which explains why the universe seems to be fine-tuned for life. If the cognized number is the numerical value of a physical variable, then the number will be subject to change in accordance with physical laws. Symmetry is a recurrent feature of the phenomenology. A mathematical formula conceived by the cognizer may also relate, one to one, the numerical values of quantities in one set with the numerical values of quantities of different dimensionality in another set, which suggests that physical laws are human inventions and that causality is a pure concept of the understanding.

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Introduction

After discovering that the results of a long-term investigation into the values of all sorts of physical parameters had been subjective (Riley, 2020), the author read Immanuel Kant's *Critique of Pure Reason* (Kant, 1998) and comprehended that the value of any number, such as the numerical value of a physical parameter, is cognized in the synthesis of intuition and mathematical concept (Riley, 2022). The results of the conceptual observation of both pure and denominate numbers are shown here graphically and through mathematical equations. The essay concludes with a short discussion.

The data used include values of the fundamental physical constants (NIST, 2024), particle mass (Particle Data Group, 2024), solar system parameters (NASA, 2024) and cosmological parameters (Planck Collaboration, 2020). All data values used are stated. Central values have been used throughout; some presented values have been rounded to a smaller number of decimal places.

1. The Cognition of Numbers

1.1. The Process of Cognition

The process of cognition employed by the author makes use of a mathematical formula to map the number of interests, N, to a number, n, the value of which explains, to the satisfaction of the cognizer, the value of N. The explanation may have a physical basis but that is not essential. Commonly, the formula maps N to n so that $N = \mathrm{b}^n$, where b is a number chosen by the cognizer. The quantity b^n is a power of the base b; n is the exponent of the power. The number n takes a rational value, at least notionally, by being located within an infinitely divisible discrete numerical framework, e.g. within a framework in which numbers take integer, half-integer, quarter-integer, etc values. The formula and the numerical framework together constitute the mathematical concept that relates N and n. If N is the value of an interesting quantity, n takes an integer or low-denomination fractional value. Since rational numbers are constructed from integers, which are exact numbers, they signal the truth; in consequence of that, rational values of n communicate the truth of the explanatory mathematical concept by which they are produced, and therefore the truth of the number of interests, N. When produced from values of N that characterize interesting quantities, integer n signals the truth forcefully. The concept then often finds wider utility.

1.2. On Particle Scales

The first concept employed by the author, many years before the subjective nature of the investigation was realized, was that particle masses derive directly from the Planck mass, m_{Planck} – a 'natural' mass scale, constructed from the speed of light, the reduced Planck constant and the gravitational constant, and equal to 2.176×10^{-8} kg or 1.221×10^{19} GeV – through division[1] by

1 Particle masses are smaller than the Planck mass.

rational powers of a number, which had to be determined (Riley, 2003). In time, three such numbers were identified: π, π/2 and e, which relate to the geometry of hypothetical warped extra dimensions (Riley, 2008); two warped geometry models had been proposed a few years previously by Randall & Sundrum (1999a&b). Rational powers of π, π/2 and e were then found for all particles; integer and low-denomination fractional powers were found for interesting particles.

The masses in Planck units of the electron and the light quarks (up, down and strange) are shown as rational powers of π and e on the 'levels' of Figure 1; the level structure has been chosen by the author. Since n_1 and n_3 are in constant ratio the points lie on a straight line. The electron and up quark (the lightest quark) occupy 'superlevels', which are levels that are characterized by level-numbers that are multiples of 5; such levels are evidently reserved for very interesting particles.

The masses in Planck units of the six quarks are shown to be equal to rational powers of π and e in Figure 2. The quarks of each family (up and down; strange and charm; top and bottom) are symmetrically arranged about superlevels in 'partnerships'. In general, if two objects of similar mass are judged to be closely related then they will be found to be arranged symmetrically in terms of the concept. Interestingly, when the up and down quarks had not been considered as partners they were not found to be symmetrically arranged (Figure 1).

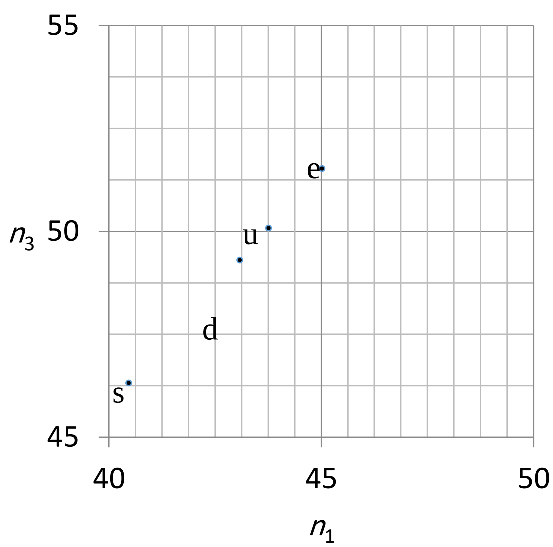

Figure 1. Points (n_1, n_3) representing the masses of the electron and the light (up, down and strange) quarks in Planck units. Each particle mass had first been expressed as $\pi^{-n_1}\, m_{\text{Planck}}$ and $e^{-n_3}\, m_{\text{Planck}}$.
electron: 0.511 MeV up quark: 2.16 MeV down quark: 4.70 MeV strange quark: 93.5 MeV

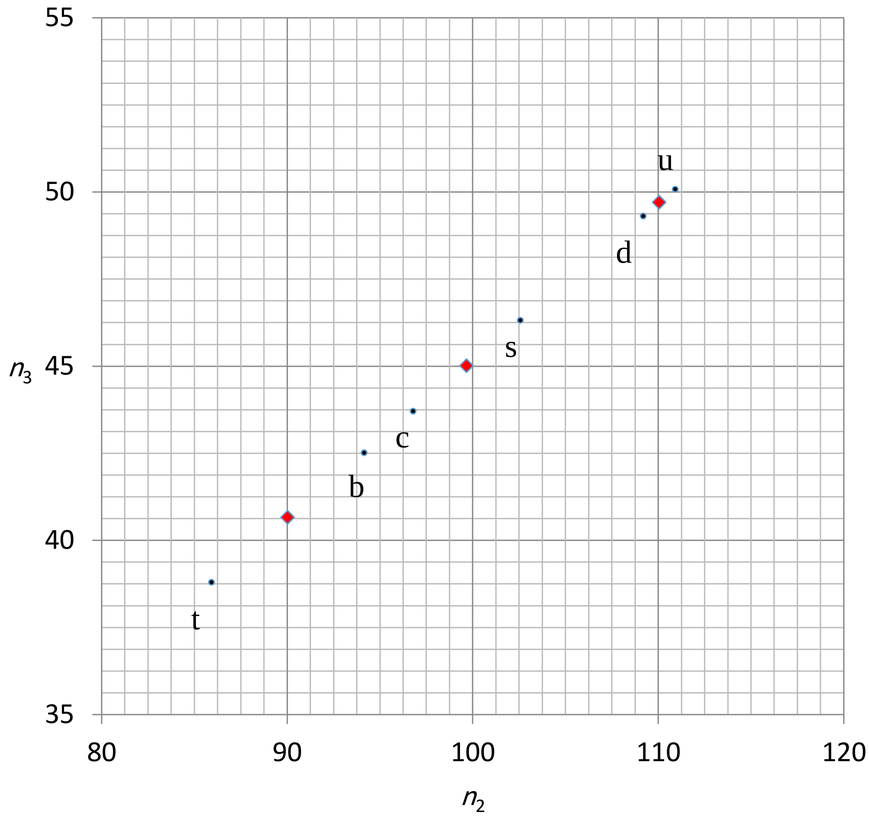

Figure 2. Points (n_2, n_3) representing the masses of the six quarks in Planck units. Each quark mass had first been expressed as $(\pi/2)^{-n_2}\ m_{\text{Planck}}$ and $e^{-n_3}\ m_{\text{Planck}}$. Diamonds mark the points of mean n_2 and n_3 for each pair of quarks: u-d, s-c and b-t.
up: 2.16 MeV down: 4.70 MeV strange: 93.5 MeV charm: 1.2730 GeV
bottom: 4.183 GeV top: 172.57 GeV

Knowing that the (spin-½) muon and the (spin-0) pseudoscalar mesons $\pi^{\pm}$, and also that the (spin-½) tau lepton and the (spin-0) pseudoscalar mesons $D^{\pm}$, are close in mass, and with supersymmetry[1] in mind, the author has found that each of the three charged leptons (electron, muon and tau lepton) forms a partnership with pseudoscalar mesons that is characterized by spin difference ½. The three charged lepton-pseudoscalar meson partnerships are shown in Figures 3 and 4. The electron and $K^{\pm}$ mesons occupy levels whose level-numbers are multiples of 3, and the particles are arranged symmetrically about an intersection of levels characterized by level-numbers that are multiples of 3. The muon and $\pi^{\pm}$ mesons, and the tau lepton and $D^{\pm}$ mesons, are arranged

1 A symmetry between particles with integer spin and those with half-integer spin.

symmetrically about levels whose level-numbers are multiples of 3, and the two partnerships are arranged symmetrically about an intersection of levels whose level-numbers are multiples of 3. Evidently, levels characterized by level-numbers that are multiples of 3 are sometimes (in addition to levels whose level-numbers are multiples of 5) considered by the cognizer to be superlevels and as such are populated by very interesting particles.

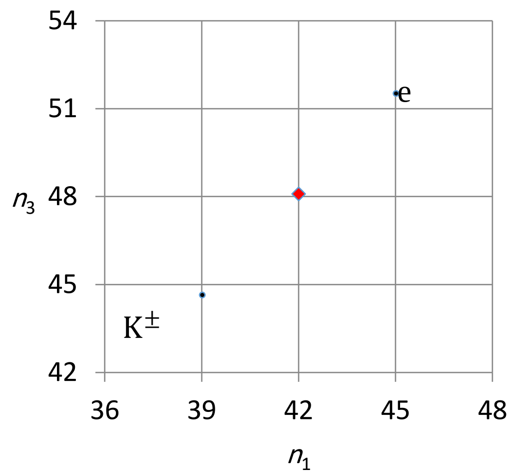

Figure 3. Points (n_1, n_3) representing the masses of the electron and the $K^{\pm}$ mesons (493.68 MeV) in Planck units. Each particle mass had first been expressed as $\pi^{-n_1}\ m_{\text{Planck}}$ and $e^{-n_3}\ m_{\text{Planck}}$. A diamond marks the point of mean n_1 and n_3.

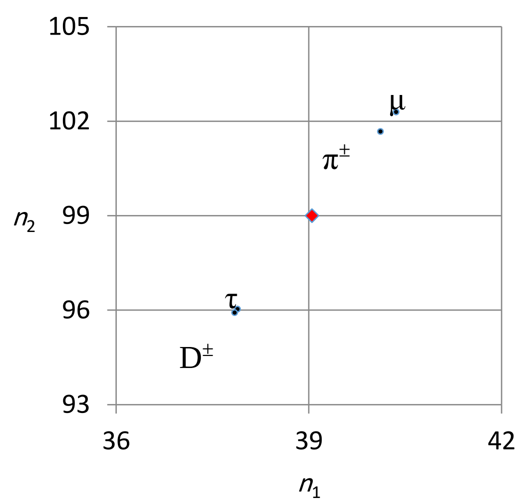

Figure 4. Points (n_1, n_2) representing the masses of (i) the muon, μ (105.66 MeV), and the $\pi^{\pm}$ mesons (139.57 MeV) and (ii) the tau lepton, τ (1776.93 MeV), and the $D^{\pm}$ mesons (1869.66 MeV), in Planck units. Each particle mass had

first been expressed as $\pi^{-n_1}\, m_{\text{Planck}}$ and $(\pi/2)^{-n_2}\, m_{\text{Planck}}$. A diamond marks the point of mean n_1 and n_2 for the four masses

Immediately on turning to length scales, the following expression was found for the Bohr radius – a physical constant that provides a measure of the size of the hydrogen atom – in terms of the Planck length, 1.616×10^{-35} m (Riley, 2013). Here and throughout the essay all figures in the exponents of powers, e.g. in the number 125.00 below, are significant.

Bohr radius (5.292×10^{-11} m)	$(\pi/2)^{125.00}\, l_{\text{Planck}}$	(1)

It follows from the value of the Bohr radius in Planck units that the mass of the electron is given by the following expression, where α is the fine structure constant (=1/137.036).

electron mass (0.511 MeV)	$\alpha^{-1}(\pi/2)^{-125.00}\, m_{\text{Planck}}$	(2)

Imagining that the masses of the three up-type quarks (up, charm and top) are related to the mass of the electron through symmetry, the expressions shown below were discovered.

up quark mass (2.16 MeV)	$\alpha(\pi/2)^{-100.0}\, m_{\text{Planck}}$	(3)
charm quark mass (1.27 GeV)	$\alpha^{2}(\pi/2)^{-75.0}\, m_{\text{Planck}}$	(4)
top quark mass (172.57 GeV)	$\alpha(\pi/2)^{-75.0}\, m_{\text{Planck}}$	(5)

The following expression has been found for the mass of the Higgs boson. The power of 2 is exact.

Higgs boson mass (125.20 GeV)	$2^{25}(\pi/2)^{-125.00}\, m_{\text{Planck}}$	(6)

The following expressions have been found for the masses of the up-type quarks in kilograms.

up quark mass (2.16 MeV)	$(\pi/2)^{-150.0}$ kg	(7)
charm quark mass (1.27 GeV)	$\alpha(\pi/2)^{-125.0}$ kg	(8)
top quark mass (172.57 GeV)	$(\pi/2)^{-125.0}$ kg	(9)

The following expression for the kilogram was then known.

Kilogram	$\alpha(\pi/2)^{50.0}\, m_{\text{Planck}}$	(10)

Arresting expressions have also been found for the meter and second in Planck units.

Meter	$\pi^{70.0}\, l_{\text{Planck}}$	(11)
Second	$e^{100}\, t_{\text{Planck}}$	(12)

Arresting expressions may be found for quantities in any appropriate unit:

W boson mass (80.37 GeV)	$(\pi/2)^{25.0}$ MeV	(13)
Z boson mass (91.19 GeV)	$\pi^{10.0}$ MeV	(14)
reduced Planck constant (ħ) (6.582×10^{-16} eV. s)	$e^{-35.0}$ eV. s	(15)
speed of light (2.998×10^{8} m. s^{-1} or 6.706×10^{8} mph)	$(\pi/2)^{45.0}$ mph	(16)

The masses in MeV of the W, Z and Higgs bosons are shown as integer powers of π and $\pi/2$ in Figure 5.

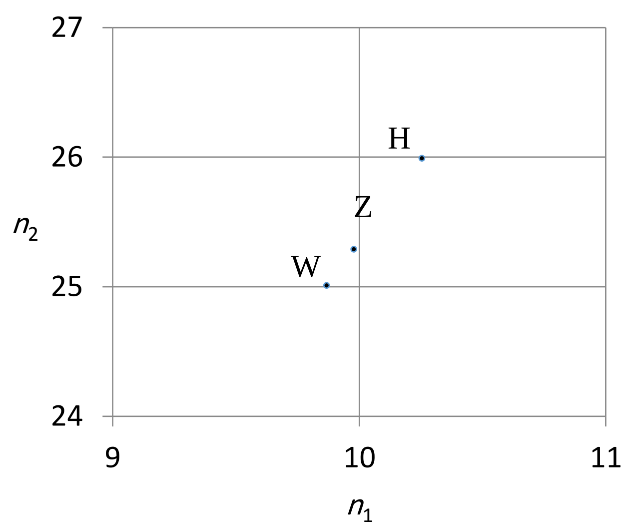

Figure 5: Points (n_1, n_2) representing the masses of the W, Z and Higgs bosons in MeV. Each particle mass had first been expressed as π^{-n_1} MeV and $(\pi/2)^{-n_2}$ MeV.

1.3. On Human Scales

Some common units besides the kilogram, meter and second have been found to have arresting values in Planck units, as shown in Figures 6, 7 and 8. The meter and yard (0.9144 m) lie at the intersection (70, 80) in Figure 6. The expression for the meter was given in (11). The expression for the yard is shown below.

Yard $\quad e^{80.0}\ l_{\text{Planck}}$ (17)

The following expression has been found for the mean calendar month (30.44 days).

Month $\quad \pi^{100}\ t_{\text{Planck}}$ (18)

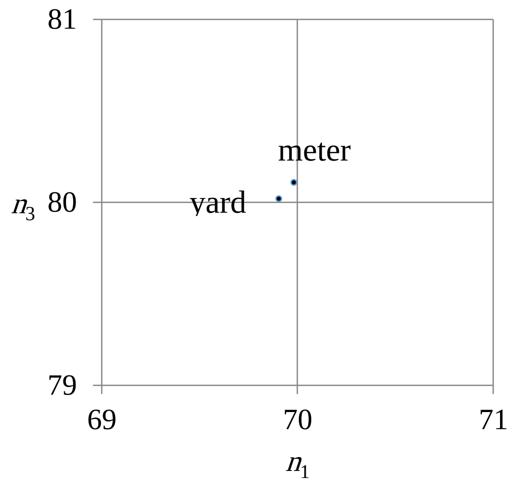

Figure 6. Points (n_1, n_3) representing the lengths of the meter and yard in Planck units. Each length had first been expressed as $\pi^{n_1}\ l_{\text{Planck}}$ and $e^{n_3}\ l_{\text{Planck}}$.

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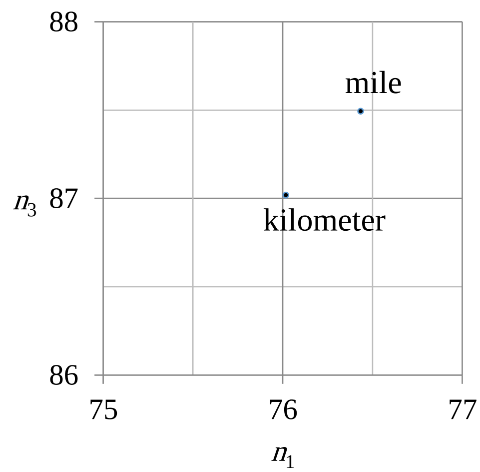


Figure 7. Points (n_1, n_3) representing the lengths of the kilometer and mile (1.609 km) in Planck units. Each length had first been expressed as $\pi^{n_1}\ l_{\text{Planck}}$ and $e^{n_3}\ l_{\text{Planck}}$.

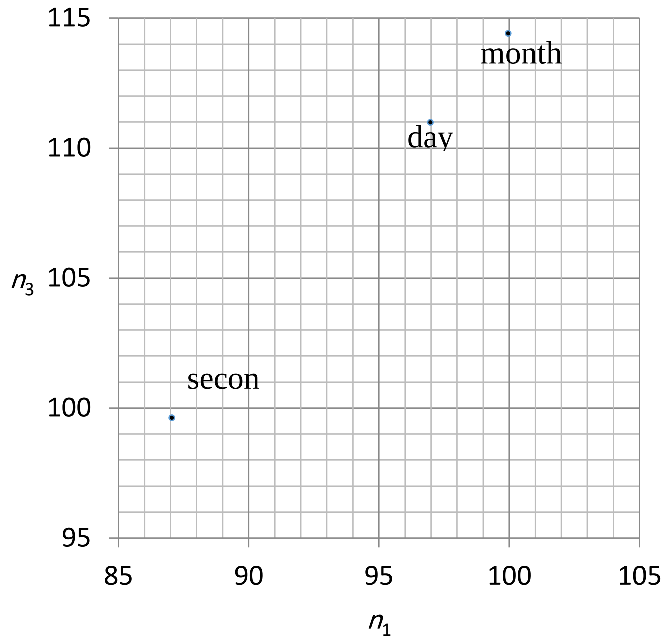


Figure 8. Points (n_1, n_3) representing the lengths of time of the second, day and mean calendar month (30.44 days) in Planck units. Each length of time had first been expressed as $\pi^{n_1}\ t_{\text{Planck}}$ and $e^{n_3}\ t_{\text{Planck}}$.

1.4. On Astronomical Scales

The masses, first in Planck units and then in kilograms, of various astronomical objects are shown as rational (integer in most cases) powers of π and e in Figures 9 and 10. Sagittarius A* is the supermassive black hole located in the centre of the Milky Way. The mass of the 'very interesting' earth is $\approx \pi^{50}$ kg.

The lengths, first in Planck units and then in meters, of various astronomical distances are shown as rational (integer in most cases) powers of π and e in Figures 11 and 12. The Hubble length is the distance from earth at which galaxies are currently receding from us at the speed of light due to the expansion of space. Notice that the semi-major axis of the 'very interesting' moon's orbit, which closely approximates to its mean distance from earth, is $\approx e^{100}\ l_{\text{Planck}}$.

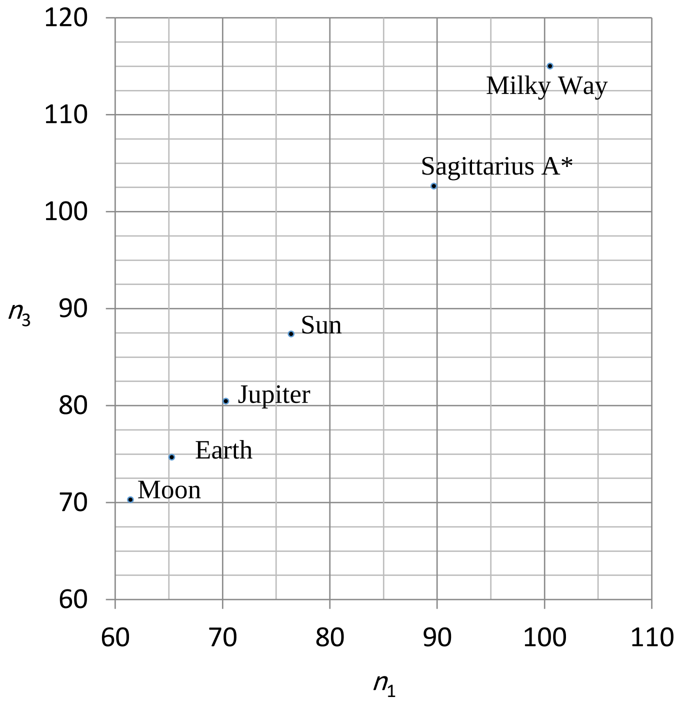


Figure 9. Points (n_1, n_3) representing the masses of six astronomical objects in Planck units. Each mass value had first been expressed as $\pi^{n_1}\ m_{\text{Planck}}$ and $e^{n_3}\ m_{\text{Planck}}$.
Moon: 7.346×10^{22} kg Earth: 5.972×10^{24} kg Jupiter: 1.898×10^{27} kg Sun: 1.988×10^{30} kg
Sagittarius A*: 8.35×10^{36} kg ($4.2\times10^{6}\ M_{\text{sun}}$) Milky Way: $\sim2\times10^{42}$ kg ($\sim1\times10^{12}\ M_{\text{sun}}$)

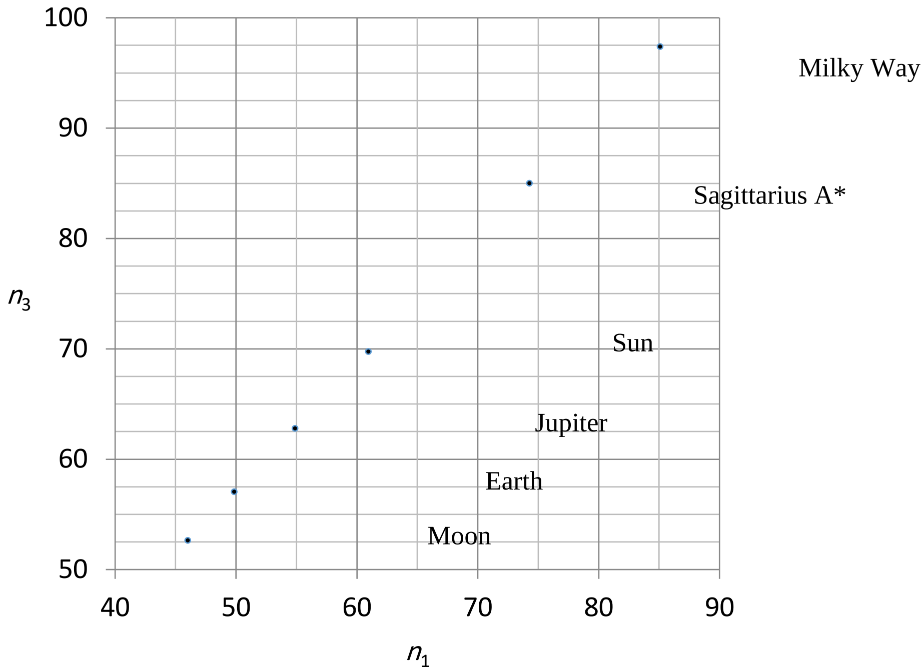


Figure 10. Points (n_1, n_3) representing the masses of six astronomical objects in kilograms. Each mass value had first been expressed as π^{n_1} kg and e^{n_3} kg. Companion to Figure 9.
Moon: 7.346×10^{22} kg Earth: 5.972×10^{24} kg Jupiter: 1.898×10^{27} kg Sun: 1.988×10^{30} kg
Sagittarius A*: 8.35×10^{36} kg (4.2×10^{6} M_{sun}) Milky Way: $\sim2\times10^{42}$ kg ($\sim1\times10^{12}$ M_{sun})

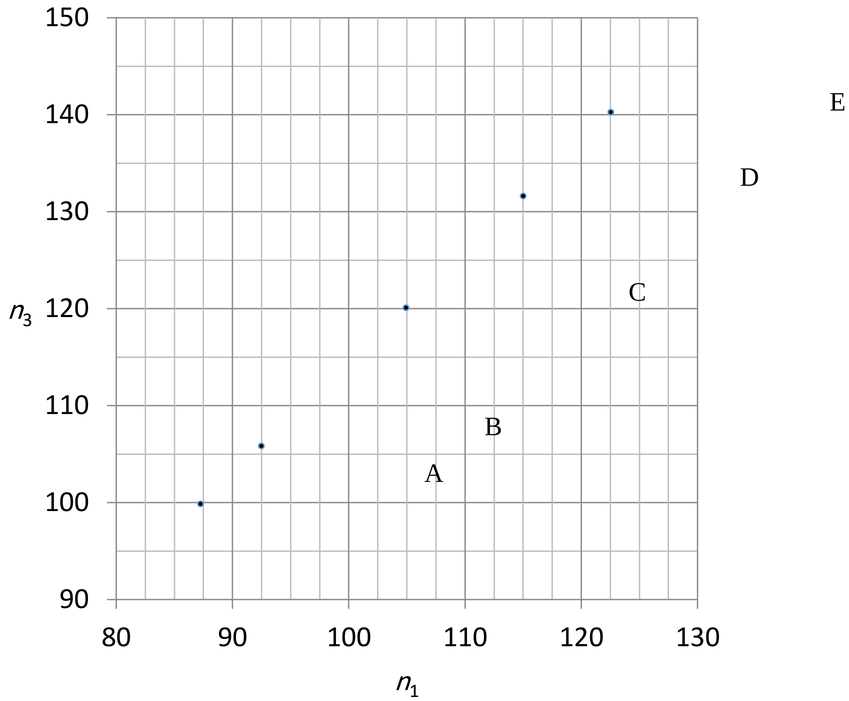


Figure 11. Points (n_1, n_3) representing five astronomical distances in Planck units. Each distance had first been expressed as $\pi^{n_1}\ l_{\text{Planck}}$ and $e^{n_3}\ l_{\text{Planck}}$ (Riley, 2020).

A Semi-major axis of the Moon's orbit (3.844×10^{8} m)
B Semi-major axis of the Earth's orbit (1.496×10^{11} m)
C Distance from Earth of the prominent star Vega (25.0 lyr)
D Distance from Earth of the Andromeda Galaxy (M31) (2.5 Mlyr)
E Hubble length (14.4 Glyr)

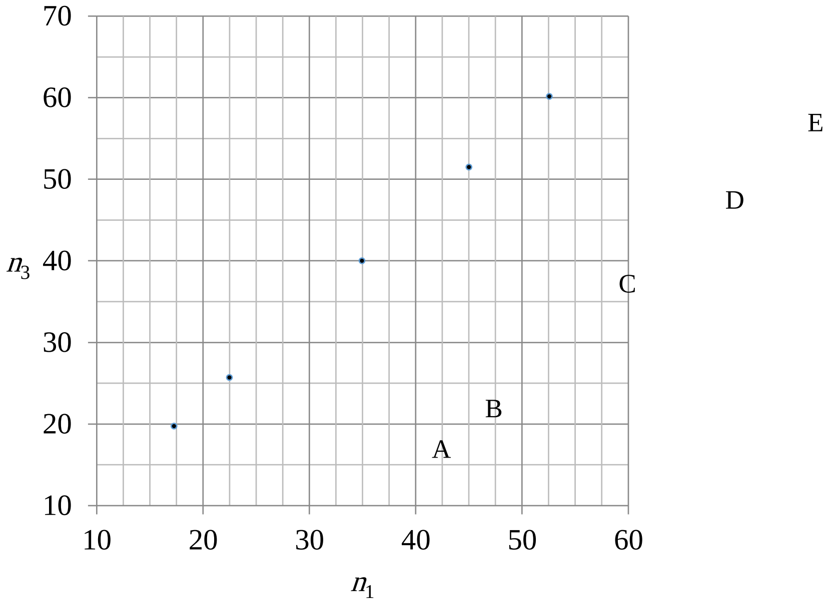


Figure 12.: Points (n_1, n_3) representing five astronomical distances in meters. Each distance had first been expressed as π^{n_1} m and e^{n_3} m. Companion to Figure 11.

A Semi-major axis of the Moon's orbit, 3.844×10^8 m
B Semi-major axis of the Earth's orbit, 1.496×10^{11} m
C Distance from Earth of the prominent star Vega, 25.0 lyr
D Distance from Earth of the Andromeda Galaxy (M31), 2.5 Mlyr
E Hubble length, 14.4 Glyr

In terms of the explanatory concept used, various parameters of the sun and moon take arresting values in Planck units, as shown below.[1]

Sun:

radius (6.957×10^5 km)	$e^{100}\ l_{\mathrm{Planck}}$	(19)
rotation period (≈27 days)	$\pi^{100}\ t_{\mathrm{Planck}}$	(20)
central temperature (1.571×10^7 K)	$\pi^{-50}\ T_{\mathrm{Planck}}$	(21)
angular momentum (1.93×10^{41} kg. m^2. s^{-1} (Riley, 2018b)	$2^{250}\ \hbar$	(22)

Moon:

[1] More precisely, $R_{\mathrm{sun}} = e^{100.47}\ l_{\mathrm{Planck}}$ and $d_{\mathrm{moon}} = e^{99.88}\ l_{\mathrm{Planck}}$

semi-major axis (3.844×10^5 km)	$e^{100}\ l_{\text{Planck}}$	(23)
orbital period (27.32 days)	$\pi^{100}\ t_{\text{Planck}}$	(24)

Certain parameters of the sun and moon are arranged symmetrically in terms of the concept, reflecting the fact that the sun dominates the sky during the day and the moon dominates the sky during the night, the two bodies thereby forming a partnership, at least in the mind of the cognizer. After expressing the radii of the sun and the moon, R_{sun} and R_{moon} (1.737×10^3 km), and the distances from earth of the two bodies, d_{sun} (1.496×10^8 km) and d_{moon}, each as $\pi^{n_1}\ l_{Planck}$ and $e^{n_3}\ l_{Planck}$, each length has been represented in Figure 13 by a point (n_1, n_3). The four points are arranged about the intersection (87.5, 100). The values of n_1 for R_{sun} and d_{moon} are symmetrically arranged about $n_1 = 87.5$, as are the values of n_1 for R_{moon} and d_{sun}. The mean value of n_1 for R_{sun} and d_{moon} is 87.51. The mean value of n_1 for R_{moon} and d_{sun} is 87.50. It follows that,

$$R_{\text{sun}} d_{\text{moon}} = R_{\text{moon}} d_{\text{sun}} \tag{25}$$

and therefore,

$$\frac{R_{\text{sun}}}{d_{\text{sun}}} = \frac{R_{\text{moon}}}{d_{\text{moon}}} \tag{26}$$

which indicates that the sun and moon are the same size in the sky. It is precisely because the sun and moon are regarded by the observer as a partnership that the process of cognition carried out by the observer results in them being of equal size in the sky.

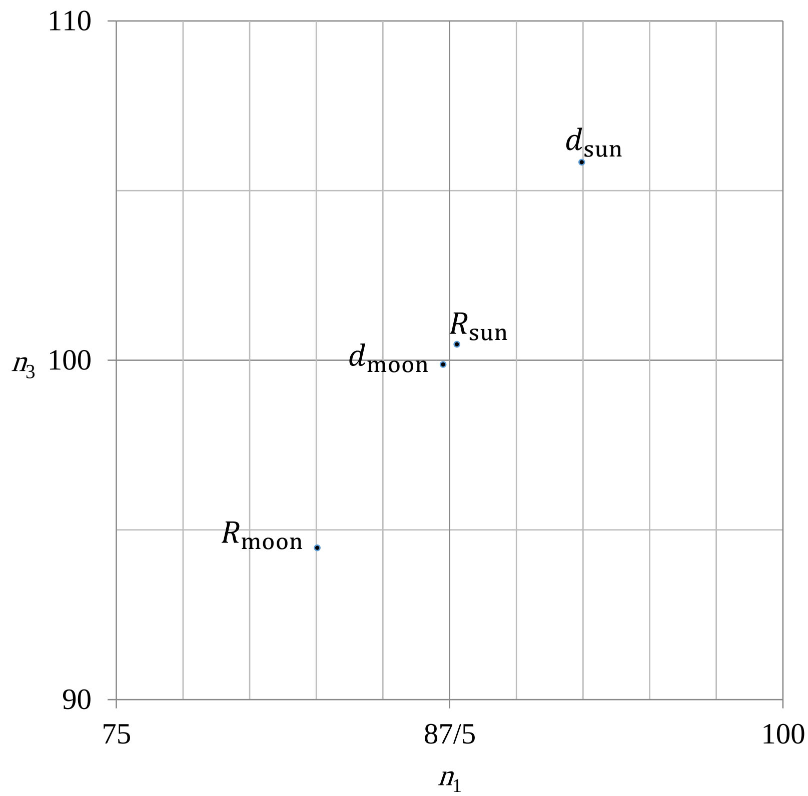


Figure 13. Points (n_1, n_3) representing the mean distances from earth (d_{sun} and d_{moon}) and the radii (R_{sun} and R_{moon}) of the sun and moon in Planck units. Each length is expressed as $\pi^{n_1}\ l_{\mathrm{Planck}}$ and $\mathrm{e}^{n_3}\ l_{\mathrm{Planck}}$.

Any concept can be used successfully in the cognition of a number. The mean orbital velocity of each planet of the solar system, in numbers of $\mathrm{km.\,s^{-1}}$, is shown to be a rational power of either π or $\pi/2$ in Figure 14. Each velocity, v in numbers of $\mathrm{km.\,s^{-1}}$, is then shown in Figure 15 to be either a rational number n_4 or a rational number n_5, where $n_4 = v(\mathrm{km.\,s^{-1}})/7.7$ and $n_5 = v(\mathrm{km.\,s^{-1}})/9.9$. The points (n_4, n_5) have been located within a unique numerical framework: the minor units are sevenths and ninths.

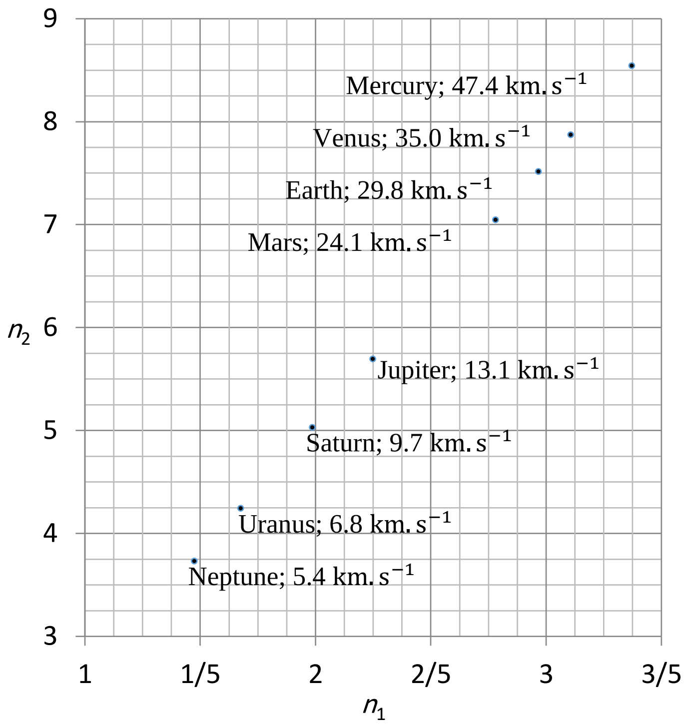


Figure 14. Points (n_1, n_3) representing the mean orbital velocities of the planets in km.s^{-1}. Each velocity had first been expressed as π^{n_1} km. s^{-1} and $(\pi/2)^{n_2}$ km. s^{-1}.

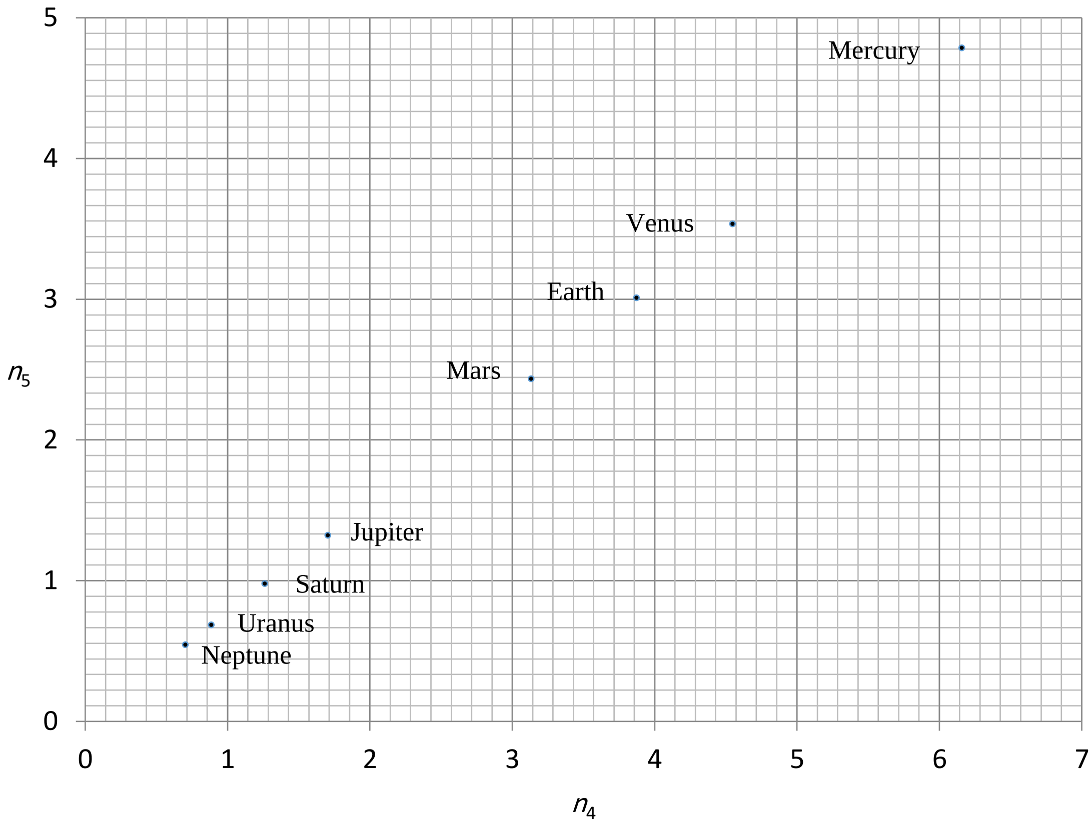


Figure 15. Points (n_4, n_5) representing the mean orbital velocities of the planets in km.s^{-1}. Each velocity, v, had first been expressed as $n_4 = v(\mathrm{km.s^{-1}})/7.7$ and $n_5 = v(\mathrm{km.s^{-1}})/9.9$. Companion to Figure 14.

1.5. On Cosmic Scales

The total mass + energy content of the (critically dense) observable universe of radius 46.6 Glyr and Hubble constant 67.4 km. s^{-1}. Mpc^{-1} is of mass equivalent 3.06×10^{54} kg. In Planck units, this quantity can be expressed as follows:

mass + energy of the observable universe (mass equivalent) $\quad \pi^{125.0}\, m_{\mathrm{Planck}}$ (27)

With a dark energy density parameter, Ω_Λ, of 0.685 (Planck Collaboration, 2018) the dark energy content can be expressed as follows.

dark energy of the observable universe (mass equivalent) $\quad \mathrm{e}^{125.1}$ kg (28)

Both of these 'very interesting' quantities (total mass + energy and dark energy content) have very interesting numerical values.

1.6. On Assorted Scales

Employing a new concept (distances in meters take numerical values that are rational powers of 2.2 and 3.3, chosen arbitrarily), various distances have been found (in succession and in the same order as presented in Figure 16) to take interesting values (Riley, 2021). Bursts of discovery such as this can occur with a new concept.

In Figure 16(a), the distance from earth of the Large Magellanic Cloud takes the value $(2.2)^{62.00}$ m, while the distances of the Large and Small Magellanic Clouds – a *pair* of interesting objects (dwarf galaxies in orbit around the Milky Way) – are symmetrically arranged about the value $(3.3)^{41.01}$ m. The two objects had been judged to form a partnership.

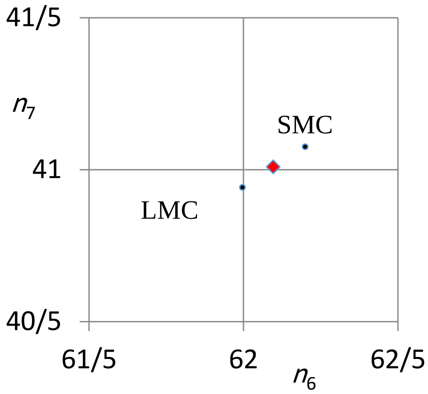
That the Bohr radius (Figure 16(f)) takes the values $(\pi/2)^{125.00}\ l_{Planck}$ and $(2.2)^{-30.01}$ m, while the meter is of value $\pi^{69.98}\ l_{Planck}$, tells one that,

$$(\pi/2)^{125} \times (2.2)^{30} \approx \pi^{70} \tag{29}$$

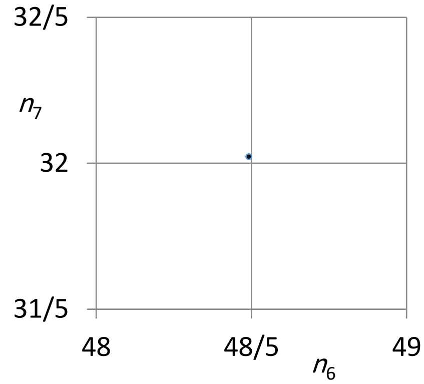
The exponent (-19.82) of the base 3.3 is also close in value to a very interesting number, i.e. -20, so that, although less precisely,

$$(\pi/2)^{125} \times (3.3)^{20} \approx \pi^{70} \tag{30}$$

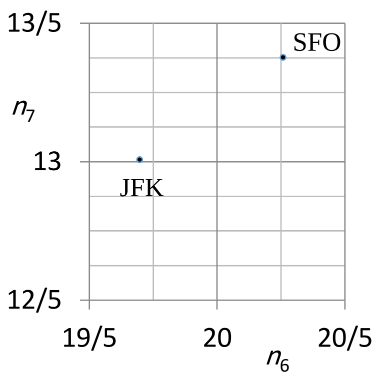
Equations (29) and (30), each of which combines three very interesting quantities, alludes to a considerable amount of iterative calculation yet the bases 2.2 and 3.3 were chosen on a whim. How the equalities in (29) and (30) might have come about will be discussed in 3.



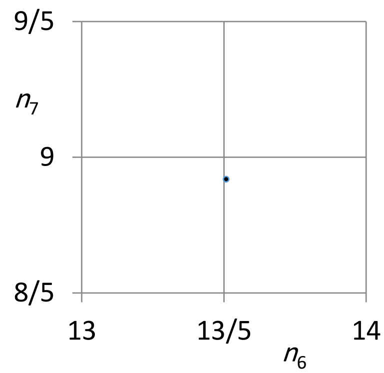
(16a) Distances from earth of the Large Magellanic Cloud (179, 000 lyr) and the Small Magellanic Cloud (210,000 lyr); (NASA, 2020). A diamond marks the point of mean n_6 and n_7.



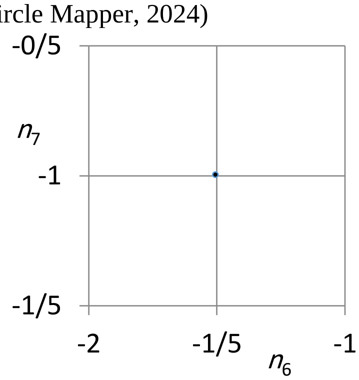
(16b) Distance from earth of Proxima Centauri (4.25 lyr)



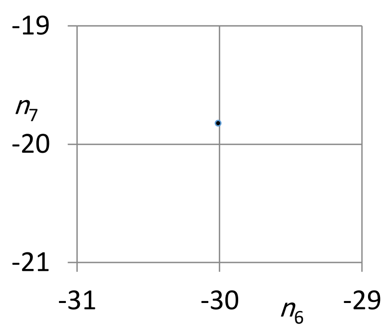
(16c) Distances from London LHR of New York JFK (5555 km) and San Francisco SFO (8638 km) (Great Circle Mapper, 2024)



(16d) The marathon distance (42.195 km)



(16e) The foot (0.3048 m)



(16f) The Bohr radius (5.292×10^{-11} m)

Figure 16. Points (n_6, n_7) representing assorted distances in meters. Each distance had first been expressed as $(2.2)^{n_6}$ m and $(3.3)^{n_7}$ m.

1.7. Dimensionless Numbers

A book – Brian Greene's *The Hidden Reality* (Greene, 2011) – has been analysed to determine whether there are signs of the observer's influence on what is observed of that publication. Here, the page numbers of the first references to British scientists whose names drew the attention of the observer are shown as powers of π and e in Figure 17. The exponents, n_1 and n_3, of π and e are found to align closely with low-denomination rational numbers despite the constraints imposed by a book. At first, this result, although expected on the basis of experience, was difficult to comprehend. However, there is a Kantian explanation. What the observer sees of a book is part of the phenomenal world, constructed in the unity of human sensibility and understanding, and not reality as it is in itself: the noumenal world.

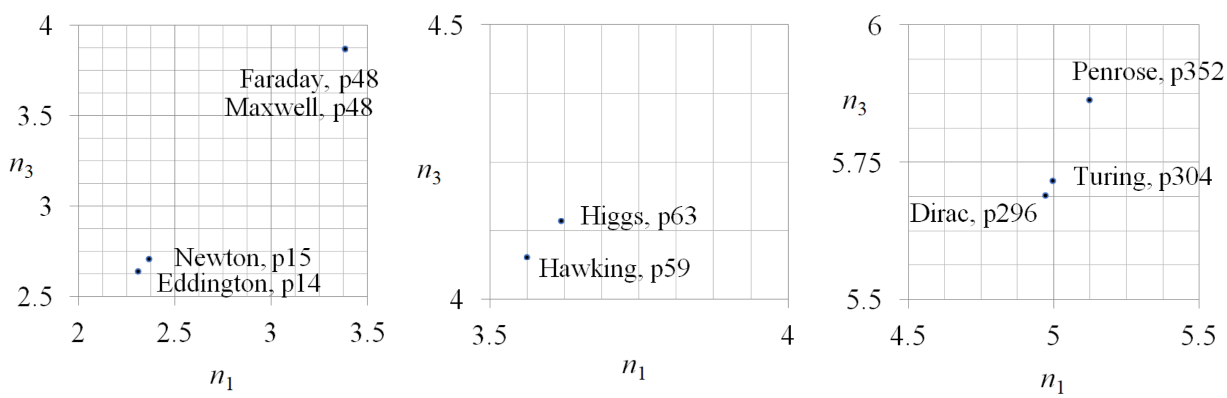


Figure 17. Points (n_1, n_3) representing the page numbers of the first references in the index of Brian Greene's *The Hidden Reality* (Greene, 2011) to British scientists whose names drew the attention of the observer. Each page number has been expressed as π^{-n_1} and e^{-n_3} .

2. Physical Laws

The precise equality,

$$2{a_0}^5 = {R_{\mathrm{OU}}}^2 \tag{31}$$

where a_0 is the numerical value of the Bohr radius measured in Planck units and R_{OU} is the numerical value of the radius of the observable universe (14.3 Gpc) measured in Planck units was found by calculating the dark energy density in two different ways and then relating the two values (Riley, 2015). The equation can be balanced dimensionally with a denominate constant (${l_{\mathrm{Planck}}}^3$). Equations of this form have subsequently been used to relate the numerical values of disparate physical variables, as follows.

The following equation of the form of (31) has been used to derive specific subatomic mass scales, m_{sa}, from the distances of stars from earth, d_* (Riley, 2020).

$$2{m_{sa}}^{-5} = {d_*}^2 \quad (32)$$

The negative exponent of m_{sa} arises because subatomic mass scales are smaller than Planck scale while all length scales are larger than Planck scale. The derived mass scales are shown as rational powers of π and e in Figure 18. Interestingly, the celebrated supergiant Betelgeuse occupies the most precise integer level intersection known, $(n_1, n_2) = (43, 109)$, which in the early years of this investigation – at the time, the light quark masses had not been measured with great precision – had been associated with the up and down quark masses (Riley, 2008).

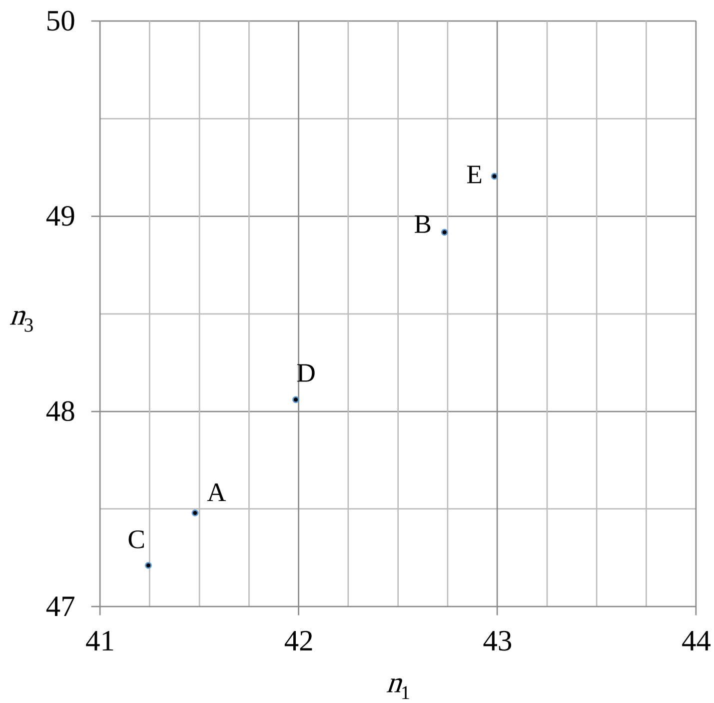


Figure 18. Points (n_1, n_3) representing subatomic mass values, m_{sa}, in Planck units, derived from the distances from earth, d_*, measured in Planck units, of the four brightest stars in the night sky and the supergiant Betelgeuse according to the equation $m_{d_*} = 2^{1/5}{d_*}^{-2/5}$. Each derived mass value has been expressed as $\pi^{-n_1}\, m_{\text{Planck}}$ and $\mathrm{e}^{-n_3}\, m_{\text{Planck}}$. The stellar distances used:

- A Sirius, the brightest star: 2.636 pc
- B Canopus, the second brightest star: 95.95 pc
- C Alpha Centauri, the third brightest star: 1.35 pc
- D Arcturus, the fourth brightest star: 11.25 pc
- E Betelgeuse: 196.9 pc

The following equation, based on (32), relates the masses of atoms, measured in Planck units, and the radii of nearby stars of similar size to the sun, measured in Planck units.

$$2{m_{atom}}^{-5} = {R_*}^2 \quad (33)$$

The radii (1.223 R_{sun} and 0.863 R_{sun}) of Alpha Centauri A and B, stars that form a binary star system, correspond to the masses of atoms of mass number 48 and 56, respectively, as shown in Figures 19 and 20. The two derived mass values are symmetrically arranged about the level $n_1 = 35$, at the intersection $(n_1, n_3) = (35, 40)$. The level $n_1 = 35$ is the domain of ^{52}Cr, the principal isotope of naturally occurring chromium.

The radius of the sun (6.957×10^5 km) and that of Tau Ceti (0.793 R_{sun}), which has been called the sun's twin, correspond to the masses of atoms of mass number 53 and 59, respectively, as shown in Figures 21 and 22. The two derived mass values are symmetrically arranged about the level $n_3 = 40$, at the intersection (35, 40). The level $\text{n}_3 = 40$ is the domain of ^{56}Fe, the principal isotope of naturally occurring iron.

An equation was next sought that would interrelate the atomic radii of atoms (Slater, 1964) and the masses of stars, symmetry with (33) being a prerequisite. In the end, the following equation was constructed; a reason had been found for the inclusion of the factor $(\pi/2)^{250}$ (Riley, 2018a).

$$2{r_{\text{atom}}}^5 = (\pi/2)^{250}{M_*}^2 \quad (34)$$

While the radius of Alpha Centauri A corresponds through (33) to the mass of an atom of mass number $\text{A} = 48$, e.g. the stable isotope ^{48}Ti, the mass (1.100 M_{sun}) of Alpha Centauri A has been found to correspond through (34) to an atomic radius of 143 pm; the empirical value for titanium is 140±5 pm. While the radius of Alpha Centauri B corresponds through (33) to the mass of an atom of mass number $\text{A} = 56$, e.g. the stable isotope ^{56}Fe, the mass (0.907 M_{sun}) of Alpha Centauri B has been found to correspond through (34) to an atomic radius of 132 pm; the empirical value for iron is 140±5 pm.

While the radius of the sun corresponds through (33) to the mass of an atom of mass number $\text{A} = 53$, e.g. the stable isotope ^{53}Cr, the mass of the sun (1.988×10^{30} kg) has been found to correspond through (34) to an atomic radius of 138 pm; the empirical value for chromium is 140±5 pm. While the radius of Tau Ceti corresponds through (33) to the mass of an atom of mass number $\text{A} = 59$, e.g. the stable isotope ^{59}Co, the mass (0.783 M_{sun}) of Tau Ceti has been found to correspond through (34) to an atomic radius of 125 pm; the empirical value for cobalt is 135±5 pm.

In summary, the radius in Planck units of the sun or of a nearby star of similar size to the sun corresponds through (33) to the atomic mass in Planck units of a specific period 4 transition metal, and the mass of the star corresponds, through (34), to the atomic radius typical of a period 4 transition metal.

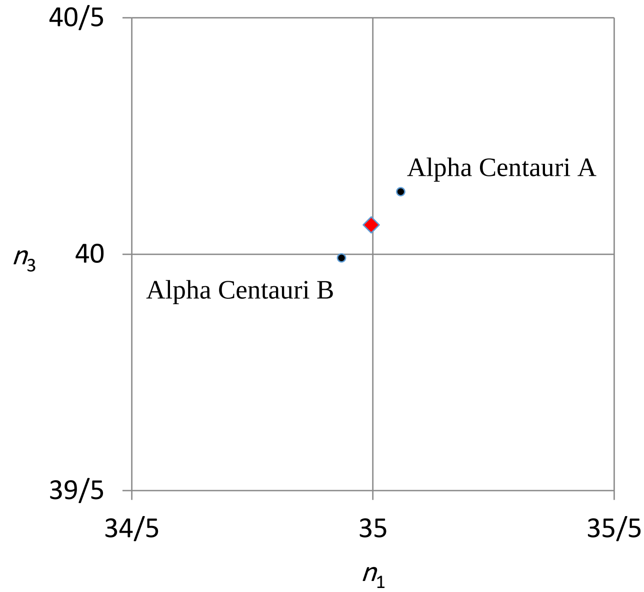


Figure 19. Points (n_1, n_3) representing mass values, m_R in Planck units, derived from the radii, R measured in Planck units, of Alpha Centauri A (1.223 R_{sun}) and B (0.863 R_{sun}) according to the equation $m_R = 2^{1/5}R^{-2/5}$. Each mass value has subsequently been expressed as $\pi^{-n_1}\ m_{\text{Planck}}$ and $e^{-n_3}\ m_{\text{Planck}}$. A diamond marks the point of mean n_1 and n_3.

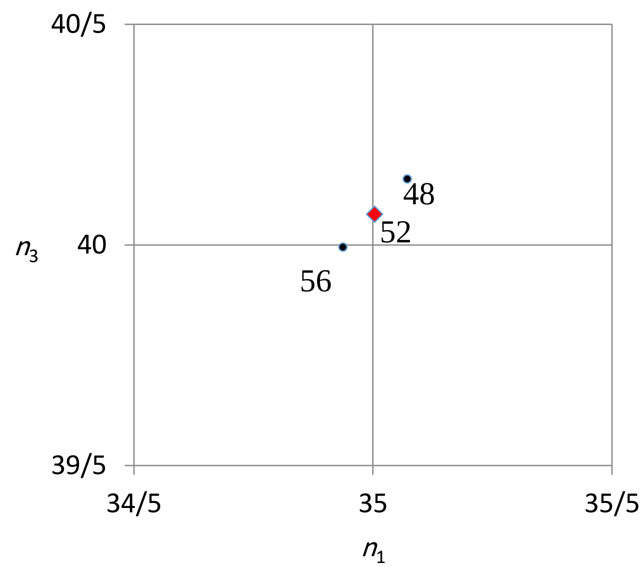


Figure 20. Points (n_1, n_3) representing the masses in Planck units of atoms of mass number A = 48 and A = 56. A diamond marks the point corresponding to A = 52. Each mass value had first been expressed as $\pi^{-n_1}\ m_{\mathrm{Planck}}$ and $\mathrm{e}^{-n_3}\ m_{\mathrm{Planck}}$. For comparison with Figure 19.

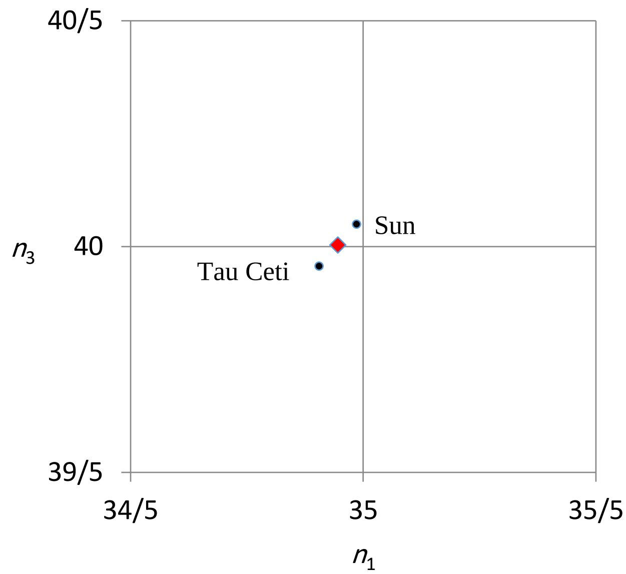


Figure 21. Points (n_1, n_3) representing mass values, m_R in Planck units, derived from the radii, R measured in Planck units, of the Sun (6.957×10^5 km) and the solar analogue Tau Ceti (0.793 R_{sun}) according to the equation $m_R = 2^{1/5}R^{-2/5}$. Each mass value has subsequently been expressed as $\pi^{-n_1}\ m_{\mathrm{Planck}}$ and $\mathrm{e}^{-n_3}\ m_{\mathrm{Planck}}$. A diamond marks the point of mean n_1 and n_3.

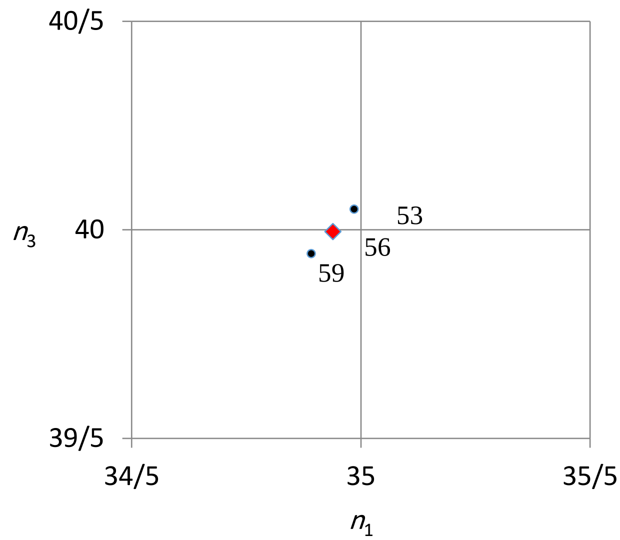


Figure 22. Points (n_1, n_3) representing the masses in Planck units of atoms of mass number $\mathrm{A} = 53$ and $\mathrm{A} = 59$. A diamond marks the point corresponding to $\mathrm{A} = 56$. Each mass value had first been expressed as $\pi^{-n_1}\, m_{\mathrm{Planck}}$ and $\mathrm{e}^{-n_3}\, m_{\mathrm{Planck}}$. For comparison with Figure 21.

The mathematical formula of (33) relates the radius of a star and the mass of an atom. Although the range of the formula is restricted to a small number of values, were it not for the absence of any notion of causation the formula would represent a physical law; a mathematical formula relates physical variables in one set to physical variables of different dimensionality in another set. Perhaps, then, physical laws come about as the inventions of observers. If that is the case causality cannot arise from experience but is an a priori concept or category of the understanding. Equations that represent physical laws are often of elegant form because they would have been conceived in that form.

After the above results were found, the thought occurred that the atomic mass and radius of a period 4 transition metal ought to be interrelated in a recognizable way. The following equality was then found to relate the numerical values of the atomic mass, m_{53}, and radius, r_{53}, in Planck units, of ^{53}Cr, the isotope that has been shown to correspond to the sun.

$$m_{53} = 2^{25.0}/r_{53} \qquad (35)$$

Encouraged by this result, which applies to period 4 metals in general although not always as precisely, the thought occurred that the Bohr radius ought to be related to the mass of an interesting object. The following equality of the form of (35) was then found, where m_{Higgs} is the mass of the Higgs boson and a_0 is the Bohr radius (Riley, 2019).

$$m_{\text{Higgs}} = 2^{25.00}/a_0 \quad (36)$$

Since $a_0 = (\pi/2)^{125.00}\, l_{\text{Planck}}$, it follows that $m_{\text{Higgs}} = 2^{25.00}(\pi/2)^{-125.00}\, m_{\text{Planck}}$, which was shown as equation (6).

3. Discussion

Time and again, one's discoveries have been found to build on what has already been discovered. For example, after finding a highly precise and meaningful (to the cognizer) equation for the Bohr radius an equation for the mass of the electron was immediately comprehended from standard physics. This equation includes the fine structure constant, α, as a factor. That factor was then found to be required in the equations for the up and top quark masses, and is present in other particle mass equations. The factor α was found after being sought when a simpler expression for the quark masses could not be found that was consistent with reality (the phenomenal world). The quantity $(\pi/2)^{25}$, or a power of that quantity, originated with the Bohr radius and features in expressions found to relate the numerical values of 'very interesting' quantities in all sorts of contexts. After finding that the masses and radii of stable period 4 transition metals are related through a simple expression, a similar and precise expression was found to relate the mass of the Higgs boson and the Bohr radius. One thing has led to another as the knowledge of the observer has broadened. The numerical values of all sorts of quantities: quantities in any units, dimensionless quantities and differences in quantities (Riley, 2003) have all been found to map to rational numbers by the application of conceptual formulae of the observer's choice. If a particularly interesting quantity, of numerical value *N*, can be accommodated consistently within the phenomenal world of the observer so that the explanatory rational number *n* takes a conspicuous value, then it will be. Integers are particularly believable and inspire confidence in the explanatory concept; they signal to others the validity of the concept.

A mapping from the number of interests, *N*, to an explanatory rational number *n* that is an integer or even a fraction of low denomination suggests that much calculation has been performed but that has not been done, at least not within the phenomenal world. It seems the cognitive process described here is the human understanding of what has been done in reality as it is in itself: the noumenal world. The same explanation will then apply to the production of the complex equalities in equations (30) and (31) and to the invention of physical laws.

The value of a number is fixed when, in cognition, the observer has found it to be true. For the author, that is when a rational concept (a mathematical formula) explains the value of the number by the value of a rational number. Examples of such a cognitive process are readily to be found in science. If the number of interests is the numerical value of a physical quantity then, when cognized, its value will be logically consistent with the numerical values of all other physical quantities in the phenomenal world, at all times. If the quantity of interest is a physical constant, then it will always have been of the value found in cognition, which is consistent with what Kant said: that time is an a priori intuition through which the experiences of the observer are framed. That explains why the universe appears to be fine-tuned for life; the conditions for life are encapsulated in the constants of nature and those constants have been given their values by human beings.

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